Distinct asymmetry in rupture-induced inelastic strain across dipping faults: An off-fault yielding model

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1. Introduction

Extensive observational studies have focused on the fault zone structure for strike-slip faults using fault-zone trapped waves [e.g., Li et al., 1990; Vidale and Li, 2003; Li and Malin, 2008; Peng et al., 2003; Ben-Zion et al., 2003; Cochrane et al., 2009]. A low-velocity fault zone embedded in the surrounding medium has been identified, which is likely due to repeated damage by historic events on fault. Numerous theoretical studies incorporating the inelastic off-fault response have also been done for strike-slip faults [e.g., Andrews, 2005; Duan, 2008; Templeton and Rice, 2008; Ma, 2008] and provide great insights to understanding the observations for fault zone structure. The 3D dynamic rupture models of Ma [2008] based on a pressure-dependent yield criterion have confirmed the ‘flower-like’ fault zone structure [e.g., Ben-Zion et al., 2003; Rockwell and Ben-Zion, 2007; Cochrane et al., 2009] and widespread near-surface inelastic response [e.g., Schaff and Beroza, 2004].

The fault zone structure for dipping faults, however, has received much less attention. Very few fault-zone trapped wave studies have been carried out for dipping faults. Less known fault geometry at depth and/or the broken symmetry of fault geometry with respect to the free surface make the interpretation of trapped-wave observations difficult (Y.-G. Li, personal communication, 2008). Few theoretical studies have investigated the inelastic off-fault response for dipping faults and explored the distribution of rupture-induced irreversible deformation. Andrews et al. [2007] explored the off-fault inelasticity on a 60° normal fault at Yucca Mountain, focusing on the physical limit of ground motion.

Here I explore the distribution of inelastic strain induced by rupture propagation on a 30° reverse fault and a 60° normal fault and simulate 2D inelastic dynamic ruptures by incorporating a Mohr-Coulomb yield criterion with a finite-element method. The initial stresses in the medium increase linearly with depth. The rupture is governed by a slip-weakening friction law [Iida, 1972]. The simulations show that the inelastic zone confines narrowly to the fault at depth and increases its size as it nears the surface, which forms a skewed ‘flower-like’ fault zone structure bounded at the top by the free surface. The ‘flower-like’ structure is a direct result of the depth-dependent stress environment and the pressure-dependent Mohr-Coulomb yield criterion.

The free surface plays an important role in the distribution of inelastic strain. For both the reverse and normal faults the inelastic strain in the hanging wall is significantly larger than the footwall, creating a strong asymmetry across the fault. The inelastic strain, however, limits significantly the ground motion especially on the hanging wall, giving rise to a smaller asymmetry in ground motion on the hanging wall and footwall compared to the asymmetry in elastic solutions. In these numerical simulations there is no irreversible volumetric strain. In a more complete model allowing irreversible volumetric strain (dilatancy) the inelastic strain seen here will be manifested as rock damage accompanying modulus reduction (S. Ma and D. J. Andrews, Inelastic off-fault response and three-dimensional earthquake rupture dynamics on a strike-slip fault, submitted to Journal of Geophysical Research, 2009). These results provide important theoretical predictions for the fault-zone structure of dipping faults that can be tested by future field experiments.

2. Model

I consider a 30° reverse fault and a 60° normal fault in a homogeneous half space, representing two optimally-orientated dipping faults. The material properties are \( \rho = 2670 \text{ kg/m}^3 \), \( \alpha = 6000 \text{ m/s} \) and \( \beta = 3464 \text{ m/s} \), where \( \rho \), \( \alpha \), and \( \beta \) are the density, P- and S-wave velocities. Both faults are 18 km wide and reach the Earth surface (Figure 1). In order to calculate the inelastic response, the absolute stresses in the medium need be known. I assume that the vertical normal stress in the medium is the lithostatic
pressure minus hydrostatic pore pressure with the water level at the free surface, i.e., \( \sigma_{zz} = (16.37 \text{ MPa/km}) z \) (compression being positive). The other two stress components are chosen as \( \sigma_{xx} = 2.2012 \sigma_{zz} \) for the reverse fault, \( \sigma_{xx} = 0.4543 \sigma_{zz} \) for the normal fault, and \( \sigma_{zz} = 0 \) for both faults. The rotation of these stresses onto fault gives the initial friction \( \mu_0 \) (the ratio of initial shear and normal tractions) on both faults to be 0.4.

[7] I use a slip-weakening fault friction law [Ida, 1972]. The static friction \( \mu_s \) is 0.6. The dynamic friction \( \mu_d \) is 0.3 except that at both ends of the fault I taper linearly the dynamic friction from 0.3 to 0.6 over a 3 km fault width such that the rupture will absorb energy to break the surface and stop smoothly at depth. The S ratio [Andrews, 1976] \( S = (\sigma_{zz-\Delta z} - \sigma_{zz+\Delta z})/2 \) is large enough such that the rupture velocity is subcritical. Given these frictional parameters the stresses and stress drop on both faults are all depth-dependent (Figure S1 of the auxiliary material).³

[8] For the parameters in the Mohr-Coulomb yield criterion, I choose the internal friction \( \tan \varphi = 0.75 \), which is larger than the static friction 0.6 so that the fault represents a plane of weakness. Two cohesion values are used: 0 and 5 MPa, in order to investigate the sensitivity of inelastic strain to cohesion. The 5 MPa cohesion used here is representative of weathered rocks. For each fault I also carry out an elastic calculation corresponding to infinite cohesion. All the calculation parameters are summarized in Table 1.

[9] A 2D finite-element method is used to model both the dynamic rupture and wave propagation. The finite-element method uses 4-node quadrilateral elements with one-point integration scheme and hourglass controls, which is very efficient in handling material yielding. The 3D version of the code has been used by Ma [2008]. A structured finite-element mesh is used to discretize the medium while keeping the element size on fault to be 20 m. The simulations are run for 15 s. The time step is 0.001 s for the reverse fault and 0.002 s for the normal fault. The split-node scheme [Day et al., 2005] is used to model rupture dynamics. The fault is allowed to open when the normal traction becomes tensile.

3. Results

[10] I nucleate the rupture by forcing it to propagate at 2000 m/s from a point at 12 km down dip. The rupture propagates spontaneously shortly afterward. The rupture propagates predominantly up dip with the decreasing stress drop. Due to the broken symmetry of fault geometry with respect to the free surface the wave reflections from the free surface interact strongly with evolving stress on the fault, resulting in a time-dependent normal stress on the fault, which plays an important role in rupture dynamics [e.g., Oglesby et al., 1998; Ma and Beroza, 2008]. The induced inelastic off-fault response also affects significantly the rupture propagation [e.g., Andrews, 2005; Templeton and Rice, 2008].

[11] In the simulations, I allow the material to yield whenever the stresses in the medium violate the Mohr-Coulomb yield criterion. I define the scalar measure of accumulated inelastic strain over time as

\[
\eta(t) = \int_0^t d\eta,
\]

where \( d\varepsilon_{xx}^p \), \( d\varepsilon_{zz}^p \), and \( d\varepsilon_{xz}^p \) are the inelastic strain increments at one time step. The quantity \( \eta \) is different from the commonly used measure of inelastic strain \( \varepsilon^p = \sqrt{(d\varepsilon_{xx}^p)^2 + (d\varepsilon_{zz}^p)^2} \) [e.g., Andrews, 2005] in that the latter can decrease with time near the surface in these simulations, as noted by Ma [2008]. The two quantities are equivalent at depth. In the simulations I assume that yielding occurs in shear only and there is no inelastic volumetric strain.

[12] I show the distribution of inelastic strain for the reverse and normal faults when the medium reaches its final static equilibrium after the rupture propagation (Figure 2). Note that the scale is logarithmic. For the reverse fault, the hanging wall and footwall is in the compressional and extensional regime of rupture propagation, respectively. The inelastic strain is seen in the footwall only at depth (below approximately 1.2 km depth). The inelastic zone increases its size when it moves updip. The yielding occurs more easily as the confining pressure decreases. In the upper 1.2 km the inelastic zone broadens dramatically in both the

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Table 1. Calculation Parameters

<table>
<thead>
<tr>
<th>Calculation Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Density (( \rho ))</td>
<td>2670 kg/m³</td>
</tr>
<tr>
<td>P-wave speed (( c_1 ))</td>
<td>6000 m/s</td>
</tr>
<tr>
<td>S-wave speed (( c_3 ))</td>
<td>3464 m/s</td>
</tr>
<tr>
<td>Stresses in medium</td>
<td></td>
</tr>
<tr>
<td>( \sigma_{xx} = (16.37 \text{ MPa/km}) z )</td>
<td></td>
</tr>
<tr>
<td>( \sigma_{xx} = 2.2012 \sigma_{zz} ) (reverse fault)</td>
<td></td>
</tr>
<tr>
<td>( \sigma_{xx} = 0.4543 \sigma_{zz} ) (normal fault)</td>
<td></td>
</tr>
<tr>
<td>( \sigma_{zz} = 0 )</td>
<td></td>
</tr>
<tr>
<td>Static friction (( \mu_s ))</td>
<td>0.6</td>
</tr>
<tr>
<td>Dynamic friction (( \mu_d ))</td>
<td>0.3</td>
</tr>
<tr>
<td>Initial friction (( \mu_0 ))</td>
<td>0.4</td>
</tr>
<tr>
<td>Slip-weakening distance (( D_w ))</td>
<td>0.4 m</td>
</tr>
<tr>
<td>Cohesion (( c ))</td>
<td>0, 5, and 10 MPa</td>
</tr>
<tr>
<td>Internal friction (( \tan \varphi ))</td>
<td>0.75</td>
</tr>
<tr>
<td>Fault width</td>
<td>18 km</td>
</tr>
<tr>
<td>Hypocenter</td>
<td>12 km down dip</td>
</tr>
<tr>
<td>Element size on fault</td>
<td>20 m</td>
</tr>
<tr>
<td>Time step (( \text{reverse fault} ))</td>
<td>0.001 s</td>
</tr>
<tr>
<td>Time step (( \text{normal fault} ))</td>
<td>0.002 s</td>
</tr>
<tr>
<td>Time duration</td>
<td>15 s</td>
</tr>
</tbody>
</table>

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³Auxiliary materials are available in the HTML. doi:10.1029/2009GL040666.
Figure 2. The inelastic strain $\eta$ is mapped using a logarithmic scale. The inelastic zone widens as it moves up dip, forming a skewed ‘flower-like’ structure bounded at the top by the free surface. The ‘flower-like’ structure for the $30^\circ$ reverse fault is more skewed than the $60^\circ$ normal fault due to a smaller fault dip. The inelastic zone is larger with higher inelastic strain on the hanging wall than the footwall. The red dashed box denotes the region where the inelastic strain is plotted in Figure 3.

Figure 3. The inelastic strain $\eta$ is mapped using a linear scale for the region around fault tip (the red box in Figure 2). The strong asymmetry in inelastic strain is clearly seen across the fault. As cohesion increases the inelastic strain near the surface and in the footwall is reduced more significantly.
hanging wall and footwall due to a smaller confining pressure, which forms a highly skewed ‘flower-like’ structure bounded at the top by the free surface. The inelastic strain is larger and broader in the hanging wall than the footwall because of a larger inelastic response in the hanging wall, resulting in a reduced asymmetry in ground motion between the hanging wall and footwall compared to the elastic solution.

For the normal fault, the inelastic strain develops only in the hanging wall (extensional regime) at depth and widens significantly near the surface, again forming a skewed ‘flower-like’ structure – the skewness being less than that for the 30° reverse fault due to a larger fault dip. Near the surface, the inelastic strain is also seen in the footwall (compressional regime) due to the small confining pressure. Similar to the reverse fault, the inelastic zone in the hanging wall is larger with higher inelastic strain than the footwall. As cohesion increases from 0 to 5 MPa the inelastic strain in the footwall nearly disappears and most inelastic strain is concentrated in the hanging wall.

The strong asymmetry in inelastic strain across the fault is more clearly illustrated for the region around fault tip using a linear scale (Figure 3). The asymmetry is largely due to the asymmetry in the amplitude of dynamic stresses experienced in the hanging wall and footwall. Previous elastic simulations of dip-slip faulting [e.g., Oglesby et al., 1998; Ma and Beroza, 2008] demonstrate that the broken symmetry of fault geometry with respect to the free surface induces higher ground motion on the hanging wall than the footwall for both reverse and normal faults, which is consistent with ground-motion observations [e.g., Abrahamson and Somerville, 1996]. The tendency of larger ground motion in the hanging wall implies larger dynamic stresses. Near the surface where the confining pressure is low the dynamic wave field plays a dominant role in the distribution of inelastic strain and the mean pressure change due to rupture propagation is not important, as was shown by Ma [2008].

The occurrence of inelastic strain, however, reduces the ground motion significantly. Figure 4 compares the distributions of peak ground velocity between inelastic and elastic simulations. The ground motion decreases with decreasing cohesion (increasing inelastic strain). The reduction is significantly larger on the hanging wall than the footwall because of a larger inelastic response. Thus we see a reduced asymmetry in ground motion between the hanging wall and footwall. The larger ground motion on the hanging wall still persists for the 30° reverse fault. For the 60° normal fault, the largest horizontal ground motion changes to the footwall for the zero cohesion case. However, the more dominant vertical ground motion is still larger on the hanging wall. In Figure S2 we see that the inelastic off-fault response also reduces fault slip. The surface slip is reduced by 21% (reverse fault) and 20% (normal fault) when material response changes from elastic to inelastic (zero cohesion).

4. Conclusions

The inelastic dynamic rupture simulations for dipping faults with the Mohr-Coulomb yield criterion demonstrate that the hanging wall experiences larger and more extensive inelastic deformation than the footwall near the surface for both reverse and normal faults. The free surface induces larger dynamic stresses in the hanging wall during rupture propagation, which causes larger inelastic strains under a low confining pressure. This result is consistent with geologic observations that hanging wall rocks experience more intense damage [e.g., Brune, 2000, 2001].

The larger inelastic strain in the hanging wall, however, reduces ground motion on the hanging wall more significantly than the footwall, resulting in a reduced asymmetry on ground motion between hanging wall and footwall, although the asymmetry largely persists.

In a depth-dependent stress environment the yielding occurs more easily near the surface than at depth because of smaller confining pressure near the surface. The inelastic strain broadens as it moves up-dip toward the surface, forming a skewed ‘flower-like’ inelastic zone. Few field experiments (e.g., using fault zone trapped waves) have been done to investigate the fault zone structure of dipping faults. Although the models reported here do not include complexities such as material heterogeneity and pore fluid effects these results should provide theoretical insights into future observational experiments for imaging the fault zone structure of dipping faults (on continents and in subduction zones) and can be tested when the new data set is available.
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